A Broad-Band Microwave Circulator*

Edward A. Ohm[1] has recently shown some applications of microwave circulators, first suggested by A. G. Fox, S. E. Miller, and W. W. Mumford of the Bell Telephone Laboratories. One in particular, a circulator microwave modulator, I believe, needs some further discussion. This device is shown in Fig. 1. As shown, the energy from the oscillator travels to the port containing the modulator, where it is modulated, reflected from the short circuit, passed back through the modulator and to the output arm of the circulator. Let us consider what occurs if the modulator is a typical ferrite type modulator, like those commercially available. This modulator is shown in Fig. 2.

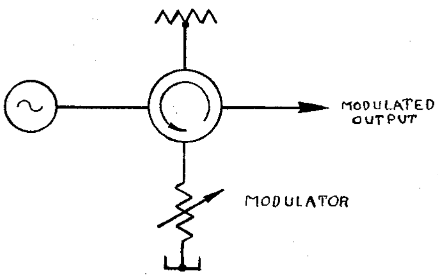

Fig. 1—A circulator microwave modulator.

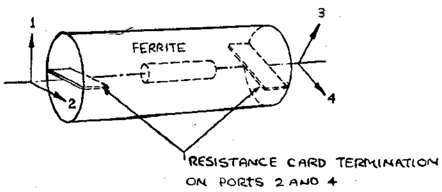

Fig. 2—Ferrite modulator.

When no field is applied to the modulator, the wave E_{incident} from the input port 1 splits between ports 3 and 4. With an applied ac field, the polarization is rotated first to port 3, the output, and then to port 4, which is terminated. The output wave at port 3 is

$$E_3 = \sin\left(\frac{\pi}{4} + m \sin \omega_m t\right) \qquad (1)$$

where m is the index of modulation and ω_m is the angular modulation frequency.

If, in general, port 3 has a load reflection coefficient Γ, then due to the nonreciprocal rotation of the ferrite, the wave reflected back out of port 1 is

$$E_1 = \Gamma E_3 \cos\left(\frac{\pi}{4} + m \sin \omega_m t\right) \qquad (2)$$

which reduces to

$$E_1 = \frac{\Gamma}{2} \sin\left(\frac{\pi}{2} + 2m \sin \omega t\right). \qquad (3)$$

A sketch of the waveforms for (1) and (3) is shown in Fig. 3.

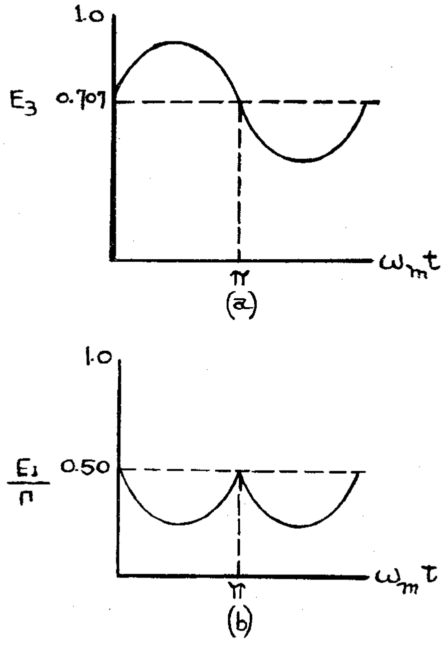

Fig. 3—Modulation envelope. (a) After passing through modulator once. (b) After being reflected from short and passing through modulator twice.

The harmonic content of (1) has been presented by Rizzi and Rich.[2] In a similar manner, the harmonic content of (3) can be analyzed; however, it is apparent that E_1 has a strong second harmonic component. Thus, the output of the circulator microwave modulator will not have good fidelity with respect to the driving voltage on the field coil. If the modulator were reciprocal instead of nonreciprocal, E_1 instead of equaling a cos sin term, would equal a $\sin^2$ term, which also has a strong second harmonic component.

From this discussion I believe it is also apparent that the use of a ferrite modulator in a microwave impedance test bench may have deleterious results, since the reflected power from the load to be measured will pull the test oscillator at approximately twice the modulation frequency. This will cause minimum filling in the standing-wave pattern and also results in a generator impedance $\neq z_0$ as a function of time.

A load isolator or attenuator should always be used with a ferrite modulator to reduce the amplitude of the wave reflected from the load.

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Author's Comment[3]

A modulator must be suited to its application and it appears that Mr. Clavin has selected a type which does not perform the task he requires. In this regard it can be shown that the fundamental modulation frequency envelope will not appear at the output of his modulator. Clavin's (3)

$$E_1 = \frac{\Gamma}{2} \sin\left(\frac{\pi}{2} + 2m \sin \omega_m t\right)$$

can be rewritten

$$E_1 = \frac{\Gamma}{2} \cos\left(2m \sin \omega_m t\right).$$

From (1) of Rizzi and Rich[2] it can be seen that this is identical to

$$E_1 = \frac{\Gamma}{2}\left[J_0(2m) + 2J_2(2m)\cos 2\omega_m t + 2J_4(2m)\cos 4\omega_m t \cdots\right]$$

and this makes it clear that E_1 does not vary at the rate of the fundamental modulation frequency.

This particular example may leave an impression that the modulation schematic shown in Fig. 6 of my paper[1] is not valid. However, since other, more successful, modulators can be designed which are in harmony with Fig. 6 it appears that some further discussion would be appropriate at this time.

One example of a more workable Faraday rotation assembly is depicted in Fig. 4. From this figure it can be shown that

$$E_{\text{reflected}} = E_{\text{in}} \cos\left(\pi/4 + 2\theta\right)$$

which is identical to

$$E_{\text{reflected}} = 0.707 E_{\text{in}}(\cos 2\theta - \sin 2\theta).$$

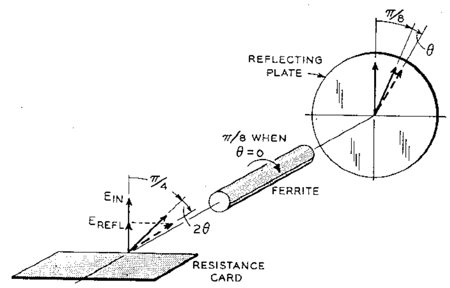

Fig. 4—A simple reflecting microwave modulator.

Now if θ varies sinusoidially at the modulation rate of ω_m with a maximum angle of rotation, $\theta_{\max}$, it can be written

$$\theta = \theta_{\max} \sin \omega_m t.$$

After inserting this value of θ, (1) of Rizzi and Rich[2] can again be used to show that $E_{\text{reflected}}$ varies at the fundamental modulation rate as well as at all of the even and odd harmonics of this rate.

$$E_{\text{reflected}} = 0.707 E_{\text{in}}\left[J_0(2\theta_{\max}) - 2J_1(2\theta_{\max})\sin \omega_m t + 2J_2(2\theta_{\max})\cos 2\omega_m t - 2J_3(2\theta_{\max})\sin 3\omega_m t + 2J_4(2\theta_{\max})\cos 4\omega_m t - \cdots\right].$$

An analysis of this equation reveals that the largest harmonic coefficients, the second and third, are down 19 db and 42 db respectively from the fundamental when the maximum value of $2\theta_{\max}$ is less than or equal to 25°. This is the value required to reduce the fundamental coefficient, $\sqrt{2}J_1(2\theta_{\max})E_{\text{in}}$, to 0.3 E_{in}, a value which corresponds to a 60 per cent modulation of a linear system.

If additional second harmonic discrimination is desired (without decreasing the fundamental component) the Faraday rota-

* Received by the PGMTT, November 13, 1956.

[1] E. A. Ohm, IRE TRANS., vol. MTT-4, pp. 210–217; October, 1956.

[2] P. A. Rizzi and D. J. Rich, "A note on sidebands produced by ferrite modulators," PROC. IRE, vol. 44, p. 556; April, 1956.

[3] Received by the PGMTT, February, 1957.

tion assembly shown in Fig. 5 can be used. Although this assembly generates slightly larger odd harmonics it is generally superior to that of Fig. 1 because it does not generate even harmonics. It can be shown that

$$E_{\text{reflected}} = E_{\text{in}} \sin^2 \phi$$

where

$$\phi = (\pi/4 + \theta_{\max} \sin \omega_m t)$$

and that this is identical to

$$E_{\text{reflected}} = \frac{E_{\text{in}}}{2} [1 + \sin (2\theta_{\max} \sin \omega_m t)].$$

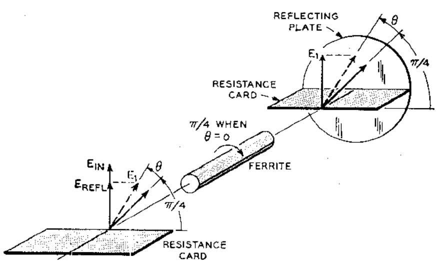

Fig. 5—A reflecting microwave modulator which does not generate even harmonics.

Again using (1) of Rizzi and Rich[2] it can be shown that the reflected voltage varies only at the fundamental and odd harmonic modulation rates.

$$E_{\text{reflected}} = E_{\text{in}}[1/2 + J_1(2\theta_{\max}) \sin \omega_m t + J_3(2\theta_{\max}) \sin 3\omega_m t + \cdots].$$

Thus, it appears that Mr. Clavin is in error when he claims that a strong second harmonic will result from a reflected voltage which varies as the $\sin^2 \phi$.

An analysis of this last equation yields data identical to that of Fig. 2 of Rizzi and Rich.[2] This figure shows how the largest harmonic coefficients, the third and fifth, vary with respect to the fundamental coefficient for different maximum values of $2\theta_{\max}$. In particular this figure shows that the third and fifth harmonics are down more than 36 db and 55 db respectively from the fundamental when the maximum value of $2\theta_{\max}$ is less than or equal to 36°, the value required to again reduce the fundamental coefficient, $J_1(2\theta_{\max})E_{\text{in}}$ to the same value of 0.3 E_{in} which corresponds to a 60 per cent modulation of a linear system.

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On Symmetrical Matching*

Mr. Mathis' note[1] is correct in that a match is achieved with the three shunt susceptances, but he is wrong in his assertion that no other positioning is possible for match. His method of matching is outlined in Fig. 1.

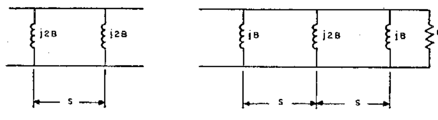

Fig. 1.

The distance s is computed so that two shunt susceptances, each of magnitude $2B$ will cancel each other,[2] thus $\tan 2\pi s/\lambda g = 1/B$. Then susceptances of magnitude B are spaced this distance away on either side and match is obtained. However, match is also obtained when $\tan 2\pi s/\lambda g = 2/B$ and thus the response is not symmetrical about the original frequency. The match can be checked by computing the admittance at the center of the network with a matched termination and, if it is purely real, the network is matched.[3]

The response can be made to be symmetrical (critically coupled) provided that the standing-wave ratios introduced by the three susceptances go in the ratios of r, r^2, r or the susceptances go as B, $B\sqrt{B^2+4}$, B. The spacing p between the three susceptances can be found on a Smith chart. See Fig. 2.

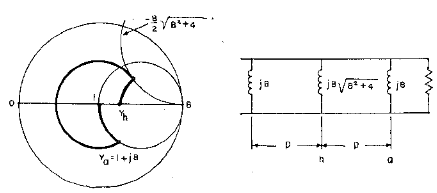

Fig. 2.

A matched termination at the right of the line makes the normalized admittance at point a $1+jB$. In Fig. 2, inductive susceptances which will have negative values are assumed. This admittance is transformed along the line toward the generator until the circle is tangent to the Smith chart circle for

$$-\frac{B}{2}\sqrt{B^2+4}.$$

The admittance at the center of the network then will be purely real and chart of the network will be the mirror image about the real axis. Therefore, the input admittance is matched. For small variations in length of line p corresponding to small changes in frequency the input admittance is still matched since the circles are tangent. This circuit is thus a critically-coupled double tuned arrangement. The value of the line length p is given by the formula.

$$\tan \frac{2\pi p}{\lambda g} = \frac{B^2 + 2 + \sqrt{B^2+4}}{B^3 + 3B}.$$

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Author's Comment[4]

Mr. Reed is correct in his remarks. My note was based on the theorem that *if one-sided matching for a lossless symmetrical discontinuity is achieved with a lossless symmetrical matching network, the matching network can be split and the part farther from the discontinuity moved to the opposite side of the discontinuity to obtain two-sided matching.* This theorem is correct, but all of the conclusions in my note were not correct.

In general, either the value of the shunt susceptances or their positions for symmetrical matching may be arbitrarily selected. When the value of the shunt susceptances is selected, there may be two pairs of positions which can be used. When the positions are selected, there may be two values of the shunt susceptances which can be used.

Procedures for finding the positions or the value of the shunt susceptances, when the other is given, are presented next. In the discussion which follows, it is assumed that the voltages, currents, and impedances are measured in units so that the characteristic impedance of the transmission line is one.

When the value B of the shunt susceptances is selected, the discontinuity is terminated in a matched load, and the input admittance Y_1 is determined and plotted on a Smith chart, as shown in Fig. 3. The admittance Y_2 given by the formula

$$Y_2 = \frac{16 + 12B^2 + 3B^4 - j(16B + 8B^3)}{16 + 12B^2 + B^4} \quad (1)$$

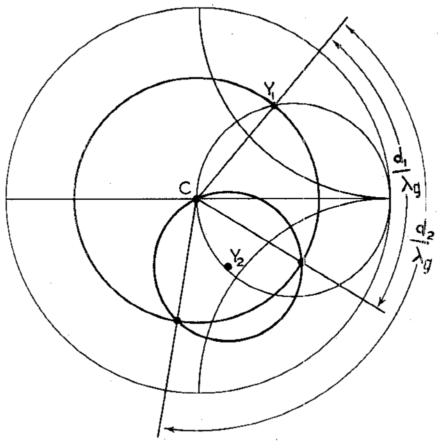

Fig. 3—Diagram for determining the positions of shunt susceptances.

is plotted on the Smith chart. A circle is drawn with its center at Y_2 which passes through the center C of the chart. (This circle must also pass through the point $Y = 1 - j2B$.) A circle is drawn with its center at C which passes through the point Y_1. The points of intersection of these circles determine the possible pairs of positions of the shunt susceptances. (For the second example given by Reed, the circles are tangent.) If the circles do not intersect, it is not possible to use this value of B. The distances d_1 and d_2 of these pairs of positions

* Received by the PGMTT, November 13, 1956.

[1] H. F. Mathis, IRE Trans., vol. MTT-4, p. 132; April, 1956.

[2] J. Reed, "Low-Q microwave filters," Proc. IRE, vol. 38, p. 794; July, 1950.

[3] *Ibid.* See (6).

[4] Received by the PGMTT, November 23, 1956.